ON SCATTERING OF A GAS BEHIND DEFLAGRATION WAVES PROPELLED BY POWERFUL RADIATION FLUXES

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Two self-similar problems about plane nonstationary gas scattering in a vacuum behind a deflagration wave for which the conservation laws are satisfied are considered. The energy flux density is considered to vary according to a power law. In the first problem the gas is transparent and scattered adiabatically. The solution is found analytically. It is shown that the Jouguet condition is conserved for a flux growing with time, while such a condition is not satisfied for a decreasing flux, and the parameters on the wave depend on the flow behind it. In the second problem, the gas absorbs radiation, where the absorption coefficient varies from infinitely large to infinitely small at some transparency temperature. The motion is isothermal. A definite fraction of the incident energy, corresponding to the effective optical thickness ~ 0.25, is extracted in the gas.

1. The surface layers of a condensed opaque substance evaporate under the effect of incident powerful radiation fluxes. In some cases, the substance being evaporated becomes transparent under the effect of optical radiation of not too high an intensity. The radiation penetrates into deeper layers of the substance, causing them to heat up, to evaporate, etc.; in substance, comparatively narrow zones of energy liberation and changes in the phase state or evaporation waves propagate [1-6] in the material.

These are deflagration waves at comparatively low radiation flux densities [7] since they are propagated at subsonic velocities relative to the material ahead of them. The parameters characterizing the state of the material ahead of and behind the wave are related by the conservation laws [3-6].

Two conditions must still be given to select the "burning" rate for a known radiation flux density resulting in a wave. One is physical: the temperature of the material behind the wave should be known. The other is gasdynamic. In the general case, this is the condition that the flow behind the wave be consistent with the law of wave propagation in a subsonic gas flow: "the overtaking characteristic yields the missing relationship" [8]. In the limit case the wave moves behind it upon compliance with the Jouguet condition. For an arbitrary law of time-variation in the radiation-flux density incident on the wave and causing it to be propagated, the law of wave motion and the flow behind it can only be found by numerical methods ([8], for example).

The case when the material behind the deflagration wave is completely transparent and is scattered adiabatically in a vacuum is often examined. The flow behind the wave is self-similar for a constant flux density and a constant velocity of wave motion (the central rarefaction wave), and the Jouguet condition is satisfied behind the wave [3-5].

If the incident radiation flux density q is variable, where it varies in time according to the power law $q \sim t^n$, and the transparency temperature and effective enthalpy of burning are invariant, then the problem is self-similar. This problem is considered herein.

Let us examine the case in which the density of the material ρ_0 ahead of the deflagration wave is large compared to the density ρ_w behind the wave. The temperature T_0 and the velocity of the material ahead of the wave u_0 are also small compared to the temperature T_w and velocity u_w behind the wave. Let us note that the pressure p_0 ahead of the wave, which is the parameter desired, is, on the other hand, greater than

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the pressure p_W behind the wave. Assuming $\rho_0 = \infty$ (or $v_0 = \rho_0^{-1} = 0$), $u_0 = 0$, $T_0 = 0$, we obtain the relationships on the wave in the form

$$p_0 = Nu_w + p_w, \quad N = \rho_w u_w, \quad NH = q_w \tag{1.1}$$

Here N is the mass flow through the wave, q_W is the radiation flux density behind the wave, H is the effective enthalpy including the heat of evaporation:

$$H = h_w + {}^1/_2 u_w^2 + Q_w \tag{1.2}$$

The enthalpy of the gas behind the evaporation wave h_W is related to p_W and ρ_W by the equation of state

$$h_w = p_w \rho_w^{-1} \gamma_w / (\gamma_w - 1) \tag{1.3}$$

where γ is the integral adiabatic index, and γ_W is its value behind the wave. Let us write the relationship between the escape velocity u_W and the sound velocity c_W:

$$|N| = M\rho_w c_w = M\sqrt{k_w p_w \rho_w} \tag{1.4}$$

Here k_W is the differential adiabatic index and the number M is a still undetermined parameter. Using (1.4), (1.1) and omitting the subscript w for convenience, we obtain

$$\begin{gathered} |u| = Mc = M\,[k(\gamma - 1)\,h / \gamma]^{1/2} \\ p_0 = p\,(1 + kM^2), \; p = q\,[h\,(\gamma - 1) / (\gamma k)]^{1/2}\,(HM)^{-1/2} \\ H = h\,(1 + {}^1/_2\,k\,(\gamma - 1)\,\gamma^{-1} M^2) + Q_v \end{gathered} \tag{1.5}$$

Let us henceforth consider that the differential and integral adiabatic indices are constant and identical. Then (1.5) simplify and become

$$\begin{gathered} |u| = M\sqrt{(\gamma - 1)\,h}, \quad p_0 = p\,(1 + \gamma M^2) \\ p = q\sqrt{(\gamma - 1)\,h} / HM\gamma, \quad H = h\,(1 + {}^1/_2\,(\gamma - 1)\,M^2) + Q_v \end{gathered} \tag{1.6}$$

For a constant value of h and unchanged value of M the ratio between the pressure p_W behind the wave [and in conformity with (1.6) the pressure p_0 ahead of the wave also] and the flux density q remains invariant. Let us note that the pressure drop p_0/p_W in a deflagration wave is small, does not exist as $M \to 0$ (essentially a subsonic heating wave), and equals ~ 2-2.67 (as γ varies between 1 and 5/3 for M=1, the Jouguet condition is satisfied).

The contribution of the kinetic energy to the effective burning enthalpy is insignificant: the fraction of kinetic in thermal energy is $(1/2)(\gamma - 1)\,M^2$, i.e., is negligible as $\gamma \to 1$ or $M \to 0$. For M=1 and $\gamma = 5/3$ it is 1/3 the thermal.

If the value of q is constant, then all the rest of the parameters are also constant for invariant M. For escape into a vacuum M=1, a central rarefaction wave is propagated behind the deflagration wave [3-5] (also see the survey papers [10, 11]).

For escape into a sufficiently low density medium or for a sufficiently high radiation flux density directly behind the deflagration wave the flow is the same as for escape into a vacuum [11, 12]. At a certain density of the medium its influence is extended to the deflagration wave and the motion becomes subsonic. The parameters behind the wave can be found by starting from the connection of the conditions behind the shock moving in the medium to the conditions on the deflagration wave [13].

The Jouguet condition is spoiled also when the material behind the wave is insufficiently transparent and a noticeable quantity of energy is liberated therein; shielding of the surface being evaporated occurs [8].

It is interesting to clarify the conditions under which the Jouguet condition is satisfied for scattering in a vacuum and for a completely transparent material behind the wave.

2. Let there be a power-law time dependence of the flux density on the wave

$$q_w = q_* \, (t / t_*)^n \tag{2.1}$$

Let us consider the magnitude of the effective enthalpy h_W, of the temperature T_W, and of the sound velocity c_W behind the wave to be invariant. Then, according to (1.6), the pressure p_W behind the wave and the density ρ_W also vary according to a power law

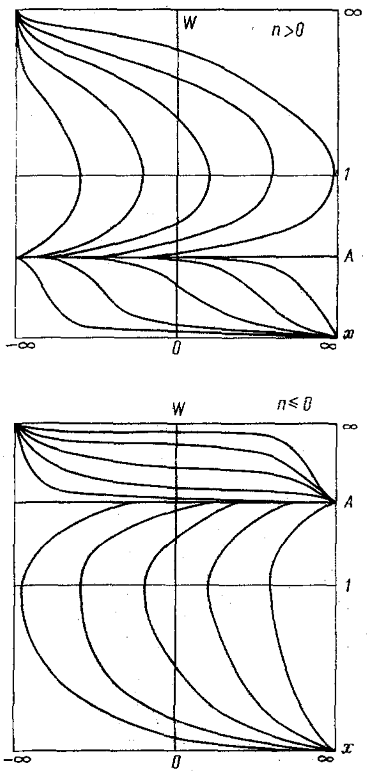

Fig. 1

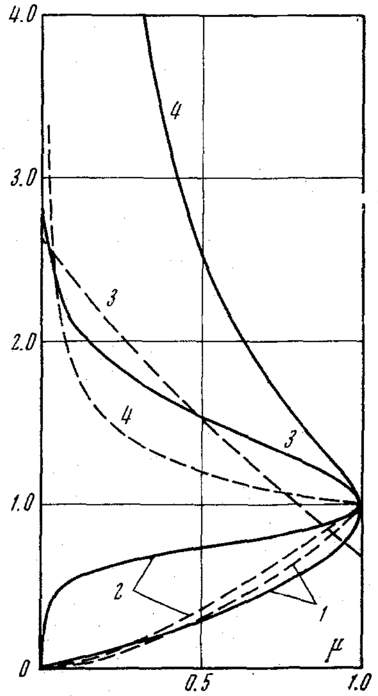

Fig. 2

$$p_w = p_* \, (t \, / \, t_*)^n, \qquad \rho_w = (\gamma p_* \, / \, c_w^2) \, (t \, / \, t_*)^n \tag{2.2}$$

Assuming M invariant, we find the law of wave motion

$$m_w = \frac{\gamma p_w M t_*}{c_w (n+1)} \, (t \, / \, t_*)^{n+1} = m_* t^{n+1} \tag{2.3}$$

The escape velocity is constant

$$u_w = c_w \tag{2.4}$$

The quantities p_* and m_* are the pressure and mass at the time $t = t_*$ when $q_W = q_*$ and are determined by means of (1.6).

The problem is solved in Lagrange mass coordinates. Let us introduce the self-similar variable

$$\mu = m \, / \, m_w = (m \, / \, m_*) \, (t \, / \, t_*)^{-(n+1)} \tag{2.5}$$

Assuming the gas behind the wave is transparent, let us consider the motion adiabatic and the entropy of distinct particles different. Starting from the relationships on the evaporation wave and the condition of entropy conservation in the particle, we obtain

$$p v^\gamma = p_* v_*^\gamma \, (t \, / \, t_*)^{-n(\gamma-1)} \, (m \, / \, m_*)^{-\lambda} \qquad \left(\lambda = \frac{n}{n+1} (\gamma - 1) \right) \tag{2.6}$$

The solution of the system of equations describing plane nonstationary gas motion will be sought as

$$p = p_* \, (t \, / \, t_*)^n P \, (\mu), \qquad v = v_* \, (t \, / \, t_*)^n V \, (\mu), \qquad u = c_w U \, (\mu) \tag{2.7}$$

We obtain a system of ordinary differential equations to determine P, V, and U:

$$\begin{gathered} -\gamma M \mu \frac{dU}{d\mu} + \frac{dP}{d\mu} = 0 \\ \frac{n}{n+1} V + \mu \frac{dV}{d\mu} + \frac{1}{M} \frac{dU}{d\mu} = 0, \qquad P V^\gamma = \mu^{-\lambda} \end{gathered} \tag{2.8}$$

The last of the relationships presented is the so-called adiabaticity integral [14, 15].

The condition on the interface with the vacuum is

$$P(0) = 0 \tag{2.9}$$

and for the parameters behind the wave, we have the conditions

$$P(1) = 1, \quad V(1) = 1, \quad U(1) = M \tag{2.10}$$

The quantity M is a still unknown parameter. Eliminating U from (2.8), we obtain

$$\frac{n}{n+1} V + \mu \frac{dV}{a\mu}\left(1 + \frac{dP}{dV} \frac{1}{\mu^2 \gamma M^2}\right) = 0 \tag{2.11}$$

For $n \leq 0$ there is an analytic solution of (2.11) satisfying (2.7)-(2.10):

$$P = \mu^k, \quad V = \mu^{-s}, \quad U = M + \frac{k}{(k-1)\gamma M}(1 - \mu^{k-1}) \tag{2.12}$$

where the exponents k and s are defined by the relationships

$$k = \frac{2\gamma}{\gamma+1}\left(1 - \frac{\gamma-1}{2\gamma}\frac{n}{n+1}\right), \quad s = \frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}\frac{n}{n+1}\right) \tag{2.13}$$

and the parameter M is defined uniquely by means of n and γ

$$M^2 = 1 + {}^1/_2 n(\gamma + 1)/\gamma \tag{2.14}$$

Therefore, for radiation fluxes decreasing with time $M < 1$, the Jouguet condition is not satisfied, the motion is subsonic, and the flow behind the deflagration wave affects its parameters.

For $n > 0$ (2.12) is not used successfully since there would be $M > 1$ according to (2.14). Let us make a change of variables

$$P = \mu^k z, \quad V = \mu^{-s} y, \quad z = y^{-\gamma}$$
$$W = y^{-(\gamma+1)} M^{-2} = z^{(\gamma+1)/\gamma} M^{-2} \tag{2.15}$$

We obtain the following equation from (2.11):

$$\frac{dW}{W}\frac{(1-W)}{(a-W)} = b\,dx \tag{2.16}$$

$$a = \left(s - \frac{n}{n+1}\right)\frac{\gamma}{k} = \left(1 + \frac{n}{2}\frac{\gamma+1}{\gamma}\right)^{-1}, \quad b = \frac{\gamma+1}{\gamma}k = 2 - \frac{\gamma-1}{\gamma}\frac{n}{n+1} \tag{2.17}$$

The boundary conditions (2.10) on the deflagration wave go over into $W(0) = W_0 = M^{-2}$. The qualitative behavior of the integral curves of (2.16) is shown in Fig. 1. The parameter W_0 has still not been determined; however, $W_0 \geq 1$ since $M \leq 1$. It is seen from Fig. 1 that all the integral curves for $W_0 > 1$ correspond to a growth in W as x diminishes. Only the integral curve passing through $W_0 = 1$ decreases. At the same time, it follows from the definition (2.15)

$$W = P^{(\gamma+1)/\gamma} \mu^{-b} \tag{2.18}$$

Since the pressure on the interface with the vacuum drops, then W should diminish as $\mu \to 0$ or as $x \to \infty$, where it will be more rapid than μ^{-b}. It is seen from (2.17) that $b > 0$. Therefore, $W_0 = M = 1$ on the deflagration wave. Therefore, subsonic flows are impossible; the Jouguet condition is satisfied for fluxes increasing with time.

Reasoning analogously for the case $n < 0$, we obtain that no integral curves are possible except $W = W_0 = a$, i.e., $M^2 = 1/a$. We hence obtain the analytic expressions (2.12) and the relationship (2.14). For $n > 0$ an analytic solution can be obtained in parametric form

$$[(W - a)/(1 - a)]^{1-a} = W\mu^{ab} \tag{2.19}$$

The distribution of the parameters in the self-similar variable μ is represented in Fig. 2 for $n > 0$ [namely, for $n = 2$ (solid curves)] and for $n < 0$ [namely, for $n = -2/3$ (dashes)] for $\gamma = 5/3$. The pressure P (curve 1), temperature PV (curve 2), and velocity U (curve 3) distributions are qualitatively similar for different n. The distributions of the specific volume V (curve 4) and the density can differ substantially for different n. For $n = 2/(\gamma + 1)$ we obtain from (2.12)-(2.14).

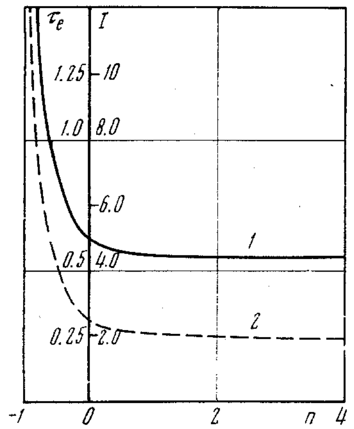

Fig. 3

$$V = 1, \quad P = PV = \mu^2, \quad U = M\left[1 + \frac{2}{\gamma - 1}(1 - \mu)\right], \quad M^2 = \frac{\gamma - 1}{\gamma} \tag{2.20}$$

Therefore, the density in the layer is constant. As n diminishes further a lower density is achieved behind the deflagration wave than near the interface with the vacuum. The terminal particle hence has an infinitely high density. Such a nature of the density distribution is related to the fact that the particles adjoining the boundary with the vacuum proceed through the deflagration wave at those times when the density on the wave would be quite high (for $n < 0$ we have infinitely high radiation flux densities, and pressure and density of the material for $t = 0$).

It is not expedient to consider solutions with $n < -1$ since they correspond to infinitely high energy delivered to the wave.

Let us examine the magnitude of the pressure p_0 ahead of a deflagration wave. According to (1.6), the pressure drop p_0/p_W in the wave diminishes as M diminishes in a subsonic flow, but the pressure behind the wave p_W rises. Consequently, the pressure p_0 ahead of the wave is increased somewhat as compared to the case when $M = 1$ (for the same flux density); however, this increase is insignificant. In the limit case we have $M^2 = (\gamma - 1)/2\gamma$ for $n = -1$. Therefore, for $\gamma = 5/3$ we have $M = 1/\sqrt{5} = 0.445$. The ratio p_0/p_W hence grows just 1.45-fold as compared with the case $M = 1$.

An approximate method to determine the pressure p_0 on the surface of a solid for a smoothly varying function $q_W(t)$ can be based on this fact. The quantity p_0 can be estimated at each instant by means of the self-similar solution by assuming that the quantity M at each instant corresponds to the instantaneous value $n = d \ln q_W / d \ln t$.

V. I. Bergel'son used the method in [8] for a numerical computation of the problem of evaporation wave propagation and vapor motion behind it for the case of transparent vapors for a typical bell-shaped pulse. These computations verified the assumption expressed above: the pressure during the whole pulse almost exactly follows the radiation flux. This method is not applicable for an abruptly varying flux magnitude, for the origination of shielding, see [8].

3. The case of adiabatic expansion of a gas behind a deflagration wave has been considered earlier. Heating of the material occurred only within the wave (the process of increasing the transparency was considered irreversible).

Evaporation waves originate also under the effect of powerful radiation sources of complex spectrum on the material [9, 10]. However, when a large number of sufficiently hard quanta is present in the spectrum, the vapor may remain opaque even when the phase transition temperature T_V is exceeded. Only for higher temperatures do ionization and bleaching of the material set in; the mean absorption coefficient with respect to the spectrum starts to drop. In some cases this drop is sufficiently abrupt (see [9], for example) near some transparency temperature T_*. Radiation energy is liberated in a narrow zone, i.e., an ionization wave, to which the deflagration wave representations are also applicable [6]. The temperature is lowered as the material scatters according to the adiabatic law. But as soon as the temperature is lowered below T_*, intense energy liberation starts. It weakens when the temperature again reaches T_*. A certain quantity of energy is liberated in the gas for such an intense, although brief, residence in the absorption domain, and the scattering occurs nonadiabatically. It is natural to consider the limit case when the absorption coefficient varies by a jump: from an infinitely large to an infinitely small value for $T = T_*$. In this case the motion can be considered isothermal. Such a problem is considered below. The quantity of energy being liberated in the gas can be determined from the solution obtained and the corresponding diminution in the radiation flux density reaching the deflagration wave can be found.

Let us consider the limit case

$$\varkappa = \infty, \quad e < e_*; \qquad \varkappa = 0, \quad e \geqslant e_* \tag{3.1}$$

The expansion occurs isothermally. The solution for this case can be obtained from the solution presented in Sec. 2 for adiabatic scattering by performing the passage to the limit $\gamma \to 1$. We have for decreasing radiation fluxes, constant in time ($n \leq 0$),

$$P = \mu, \; V = \mu^{-1}, \quad PV = 1, \quad U = M - M^{-1} \ln \mu, \quad M^2 = 1 + n \tag{3.2}$$

For increasing fluxes (n > 0) M=1 we obtain the following parameter distributions:

$$P = \mu \sqrt{W}, \quad V = \frac{1}{\mu \sqrt{W}}, \quad g = \frac{n}{n+1}$$
$$\frac{[(W-1)/(n+1)]^g}{W} = \left(\frac{n}{n+1}\right)^g \mu^{2/(n+1)}$$
$$U = 1 - g\frac{\sqrt{n+1}}{2}\ln\left[\frac{\sqrt{n+1}+1}{\sqrt{n+1}-1}\,\frac{\sqrt{W(n+1)}-1}{\sqrt{W(n+1)}+1}\right] \tag{3.3}$$

It should be kept in mind that the radiation flux density q_W will differ from the radiation flux density q_0 incident on the interface between the material and the vacuum m=0. Since the assumption about isothermy is valid: $\partial e/\partial t = 0$, then we obtain from the energy equation by using the conservation laws on the wave

$$\frac{q_w}{q_0} = \left(1 - \frac{(1-J)\,c_w^2}{2H_w}\right)^{-1}, \quad J(n) = \int_0^1 U^2(\mu)\,d\mu \tag{3.4}$$

Here c_W is the isothermal speed of sound. For c_W the integral in (3.4) can be evaluated exactly:

$$J(n) = M^2 + 2M^{-2} + 2 = 2/(n+1) + n + 3 \tag{3.5}$$

For n=0 we have J=5.

The effective optical thickness τ_e can be introduced

$$q_w / q_0 = \exp(-\tau_e) \tag{3.6}$$

The dependence of J (curve 1) and τ_e (curve 2) on n for a typical value of the ratio $H/c_W^2 = 4$ for an evaporation wave is shown in Fig. 3. For $n \geq 0$ we have $\tau_e \approx 0.25$. Therefore, radiation absorption in the region of gas scattering being the wave is insignificant.

It should be noted that the value of τ_e is close to that obtained in [12] for an approximate solution of the self-similar problem [12, 18] for a power-law dependence of the absorption coefficient on the temperature and the density.

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